

EXERCÍCIOS DE PRIMITIVAS

1. Mostre que $F(x) = \ln(1 - x) - \ln(1 + x) + 6$ é uma primitiva de $f(x) = \frac{-2}{1 - x^2}$.

2. Primitive as seguintes funções

2.1. $x^2 + 4x^5$

2.2. $(x - 1)^2$

2.3. $x^2 - \cos(2x)$

2.4. $\frac{1}{x - 2}$

2.5. $\frac{x}{x^2 + 3}$

2.6. $\frac{1}{3x - 4}$

2.7. $x^3 - e^{5x}$

2.8. $\frac{\ln(x + 1)}{x + 1}$

2.9. $\frac{1}{(2x + 1)^2}$

2.10. $\frac{\cos x}{\sqrt{\sin x}}$

2.11. $\frac{1}{4x^2 + 1}$

2.12. $\operatorname{tg} x \ln(\cos x)$

2.13. $x \sin x$

2.14. $e^x \sin x$

2.15. $\cos x \sin x$

2.16. $\arcsin x$

2.17. $\frac{\sqrt{x}}{x(\sqrt{x} - 1)}$

2.18. $\frac{x}{e^x}$

2.19. $(e^x + 2)^2$

2.20. $(\sqrt{x} + 1)(x - \sqrt{x} + 1)$

2.21. $\left(\frac{1 - x}{x}\right)^2$

2.22. $(x + 1) \sin x$

2.23. $\left(\frac{x}{e^x}\right)^2$

2.24. $x^3 e^{x^2}$

2.25. $\ln^2 x$

2.26. $\sin^2 x$

2.27. $\frac{1}{\sqrt{x}\sqrt{1-x}}$

2.28. $\frac{x^2}{\sqrt{1-x^6}}$

2.29. $\frac{1}{x\sqrt{1-\ln^2 x}}$

2.30. $\frac{\sqrt{x-1}}{x}$

2.31. $\frac{1 - e^x}{1 + e^x}$

2.32. $(1 + \cos^2 x) \sin x$

2.33. $\frac{x + 16}{(x - 1)^2}$

2.34. $\frac{e^{2x}}{e^x + 4}$

2.35. $\frac{\sin x}{\sqrt{1 + \cos x}}$

2.36. $x\sqrt{x + 1}$

2.37. $\frac{e^{2x}}{1 + e^{4x}}$

2.38. $\frac{e^x}{e^{2x} + 2e^x + 1}$

2.39. $\frac{\ln(3x)}{x^2}$

2.40. $\frac{2x + 3}{x^2 + 3x + 7}$

- 2.41. $\sin(3 - 5x)$
- 2.42. $e^x \sec^2(e^x)$
- 2.43. $\frac{2x^2}{x^2 + 1}$
- 2.44. $\frac{1}{x^2 - 1}$
- 2.45. $\frac{1}{1 + e^{-x}}$
- 2.46. $\frac{e^{3x}}{(1 + e^{3x})^2}$
- 2.47. $\frac{\ln x}{\sqrt{1 + x}}$
- 2.48. $x^2 \operatorname{arctg} x$
- 2.49. $\cos^3 x$
- 2.50. $\sin^5 x$

3. Determine $F(x)$ tal que $F'(x) = e^x + 2x^3$ e $F(0) = 3$.

4. Determine $F(x)$ tal que $F''(x) = k$ ($k \in \mathbb{R}$), $F'(0) = 2$ e $F(0) = 3$.

Soluções:

2. Uma primitiva de cada uma das funções do enunciado é:

- 2.1. $\frac{x^3}{3} + \frac{2}{3}x^6$ 2.2. $\frac{(x-1)^3}{3}$ 2.3. $\frac{x^3}{3} - \frac{\sin(2x)}{2}$ 2.4. $\ln|x-2|$ 2.5. $\frac{\ln(x^2+3)}{2}$
- 2.6. $\frac{1}{3} \ln|3x-4|$ 2.7. $\frac{x^4}{4} - \frac{1}{5}e^{5x}$ 2.8. $\frac{1}{2} \ln^2(1+x)$ 2.9. $-\frac{1}{2(2x+1)}$ 2.10. $2\sqrt{\sin x}$ 2.11. $\frac{1}{2} \operatorname{arctg}(2x)$
- 2.12. $-\frac{1}{2} \ln^2(\cos x)$ 2.13. $-x \cos x + \sin x$ 2.14. $\frac{1}{2}e^x(\sin x - \cos x)$ 2.15. $-\frac{\cos^2 x}{2}$
- 2.16. $x \arcsin x + \sqrt{1-x^2}$ 2.17. $2 \ln(\sqrt{x}-1)$ 2.18. $-xe^{-x} - e^{-x}$ 2.19. $\frac{e^{2x}}{2} + 4e^x + 4x$
- 2.20. $\frac{2}{5}x^{5/2} + x$ 2.21. $-\frac{1}{x} - 2 \ln x + x$ 2.22. $-(x+1) \cos x + \sin x$ 2.23. $-\frac{2x^2+2x+1}{4}e^{-2x}$ 2.24. $\frac{x^2-1}{2}e^{x^2}$
- 2.25. $x \ln^2 x - 2x \ln x + 2x$ 2.26. $\frac{1}{2}(x - \sin x \cos x)$ 2.27. $2 \arcsin(\sqrt{x})$ 2.28. $\frac{1}{3} \arcsin(x^3)$ 2.29. $\arcsin(\ln x)$
- 2.30. $2(\sqrt{x-1} - \operatorname{arctg} \sqrt{x-1})$ 2.31*. $x - 2 \ln(1+e^x)$ 2.32. $-\cos x - \frac{\cos^3 x}{3}$ 2.33. $\ln(x-1) - \frac{17}{x-1}$
- 2.34. $e^x - 4 \ln(e^x + 4)$ 2.35. $-2\sqrt{1+\cos x}$ 2.36. $\frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3}$ 2.37. $\frac{1}{2} \operatorname{arctg}(e^{2x})$
- 2.38. $-\frac{1}{e^x+1}$ 2.39. $-\frac{1}{x}(\ln(3x)+1)$ 2.40. $\ln(x^2+3x+7)$ 2.41. $-\frac{1}{5} \cos(3-5x)$ 2.42. $\operatorname{tg}(e^x)$
- 2.43. $2(x - \operatorname{arctg} x)$ 2.44. $\frac{1}{2}(\ln|x-1| - \ln|x+1|)$ 2.45. $\ln(e^x+1)$ 2.46. $-\frac{1}{3(1+e^{3x})}$
- 2.47. $2\sqrt{1+x}(\ln x - 2) - 2 \ln(\sqrt{1+x}-1) + 2 \ln(\sqrt{1+x}+1)$ 2.48. $\frac{x^3}{3} \operatorname{arctg} x - \frac{1}{6}x^2 + \frac{1}{6} \ln(x^2+1)$
- 2.49. $\sin x - \frac{1}{3} \sin^3 x$ 2.50. $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x$
3. $F(x) = e^x + \frac{x^4}{2} + 2$ 4. $F(x) = \frac{k}{2}x^2 + 2x + 3$

* corrigido em 04/04/2019