Resolução e discussão de sistemas lineares

1. Indique o conjunto de soluções dos sistemas lineares

$$\text{a)} \left\{ \begin{array}{rcrcrcr} x & + & y & + & z & = & 1 \\ 2x & + & y & + & z & = & 1 \\ 3x & + & y & + & z & = & 1 \end{array} \right.$$

$$\text{b)} \left\{ \begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & - & x_4 & = & 8 \\ 3x_1 & - & 6x_2 & + & 2x_3 & & & = & 18 \\ & & & & x_3 & - & 2x_4 & = & 5 \end{array} \right.$$

2. Discuta os sistemas $Ax = b$ para $\alpha, \beta \in \mathbb{R}$:

$$\text{a)}\ A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 3 & 1 & \alpha \end{bmatrix} \text{ e } b = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}.$$

$$\text{b)}\ A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \\ \alpha & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \text{ e } b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \\ \beta \end{bmatrix}.$$

$$\text{c)}\ A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 4 & 10 & \alpha \end{bmatrix} \text{ e } b = \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}.$$

$$\text{d)}\ A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & \alpha & 5 \\ 1 & 1 & 2 \end{bmatrix} \text{ e } b = \begin{bmatrix} 0 \\ \beta \\ 1 \end{bmatrix}.$$

$$\text{e)}\ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha+2 \end{bmatrix} \text{ e } b = \begin{bmatrix} \beta \\ 2\alpha \\ 0 \end{bmatrix}.$$

$$\text{f)}\ A = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \alpha & -1 \\ 1 & 1 & 1 \end{bmatrix} \text{ e } b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Soluções: **1. a)** $\{(x, y, z) : x = 0,\ y = 1 - z,\ z \in \mathbb{R}\}$ **b)** $\{(x_1, x_2, x_3, x_4) : x_1 = -2 + 2x_2,\ x_2 \in \mathbb{R},\ x_3 = 15,\ x_4 = 5\}$ **2. a)** $\alpha = 2$ Imp., $\alpha \neq 2$ PD **b)** $\beta \neq 1$ Imp. para todo o α; $\beta = 1 \left\{ \begin{array}{ll} \alpha = -5 & \text{Imp.} \\ \alpha \neq -5 & \text{PD} \end{array} \right.$; **c)** $\alpha = 14$ Ind., $\alpha \neq 14$ PD **d)** $\alpha \neq 3$ PD para todo o β; $\alpha = 3 \left\{ \begin{array}{ll} \beta = 1 & \text{Ind.} \\ \beta \neq 1 & \text{Imp.} \end{array} \right.$; **e)** $\alpha = -1 \left\{ \begin{array}{ll} \beta \neq 0 & \text{Imp.} \\ \beta = 0 & \text{Ind.} \end{array} \right.$; $\alpha = 1 \left\{ \begin{array}{ll} \beta \neq 2 & \text{Imp.} \\ \beta = 2 & \text{Ind.} \end{array} \right.$; $\alpha \neq -1, 1$ PD para todo o β **f)** $\alpha = -1$ Imp., $\alpha = 1$ Ind., $\alpha \neq -1, 1$ PD.