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Revisiting reservoir storage—yield relationships using a global streamflow database

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Abstract

Annual and monthly streamflows for 729 rivers from a global data set are used to assess the adequacy of five techniques to estimate the relationship between reservoir capacity, target draft (or yield) and reliability of supply. The techniques examined are extended deficit analysis (EDA), behaviour analysis, sequent peak algorithm (SPA), Vogel and Stedinger empirical (lognormal) method and Phien empirical (Gamma) method. In addition, a technique to adjust SPA using annual flows to account for within-year variations is assessed. Of our nine conclusions the key ones are, firstly, EDA is a useful procedure to estimate streamflow deficits and, hence, reservoir capacity for a given reliability of supply. Secondly, the behaviour method is suitable to estimate storage but has limitations if an annual time step is adopted. Thirdly, in contrast to EDA and behaviour which are based on time series of flows, if only annual statistics are available, the Vogel and Stedinger empirical method compares favorably with more detailed simulation approaches.

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1. Introduction

This paper explores the relationship between reservoir storage and yield using monthly and annual streamflow data that cover most regions of the globe. We examine reservoir capacity as the dependent variable rather than draft (yield) because most of the procedures are formulated in this way. However, as the relationship between storage size and reservoir yield needs to be specified in a reservoir storage—yield analysis it is a straightforward exercise to interchange these two variables.

The history of reservoir storage—yield analysis is a rich one with the first important method by Rippl [36] proposed nearly 125 years ago, followed by the works of Hazen [15]

and Sudler [38]. However, it was not until the 1950s that a serious effort was made to bring some mathematical rigour into the process with the activities of Hurst [16], Moran [27] and his colleagues Gani [7], Prabhu [34], Ghosal [8], Langbein [19] and Lloyd [21]. With the introduction of stochastic data generation in the early sixties, emphasis moved back to Rippl type techniques focusing on the sequent peak algorithm [39]. During the sixties, seventies and eighties several useful critical period techniques by Alexander [5], Gould [10], and Hardison [13] were proposed as well as many empirical generalizations based on stochastic data and simulation including Gould [9], Vogel and Stedinger [44] and Phien [33]. The contributions of Pegram [30] and Buchberger and Maidment [6] are important as they provide exact solutions for specific storage-yield conditions. Considerations of reservoir performance and sustainability were also a feature of the eighties and nineties especially in the works of Hashimoto et al. [14], Loucks [22] and Simonovic [37]. Throughout this latter period Klemes's

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publications [18] provided a theoretical setting for the approaches discussed in this paper.

One impetus for this paper was the availability of a global data set of monthly and annual streamflow that had been subjected to a rigorous quality assessment and covered most regions of the world. This allowed the authors to explore how well five very different storage—yield techniques (Extended Deficit Analysis, behaviour analysis, sequent peak analysis, Vogel and Stedinger procedure and Phien's empirical method) handled the wide range of streamflow characteristics that were found in the global data set, which would provide a practical setting for the testing of the procedures.

The analysis focuses on hypothetical storages at the stream gauging station for each river and was restricted to estimating a constant annual yield (75% draft) and reliability characteristics using the Standard Operating Policy in which demand is satisfied if there is sufficient water in the reservoir, otherwise the reservoir empties. Including draft ratios other than 75% and seasonal drafts in the analysis would have extended the exercise beyond the available resources.

Following this introduction, Section 2 describes the characteristics of large reservoirs located in Australia, South Africa and the United States. Next, we outline the background theory and application for five reservoir storage—yield (S—Y) techniques that are currently used in practice. The global annual and monthly streamflow data sets used to compare and contrast the S—Y methods are described in Section 4. In Sections 5 and 6, we apply the techniques to the global data set and explore the differences among the procedures to identify their attributes and inadequacies. Summary comments are made in Section 7 and relevant conclusions are drawn in Section 8.

2. Some reservoir characteristics

The three key variables in S-Y analysis are active reservoir capacity S, draft or yield D, and reliability of draft, often expressed in terms of T, the average return period (in years) of at least one failure to supply the demand in an interval (month or year). Several measures of reservoir performance other than reliability are also used, namely vulnerability and resilience, but these tend to be of secondary importance and will not be addressed in this paper. To simplify theoretical analysis active reservoir or storage capacity, which is defined as the difference between total storage capacity at full supply level and dead storage (the volume of water held below the lowest off-take) is used. The capacity is expressed either as a ratio of mean annual inflow S/μ , or as a ratio of the standard deviation of annual inflows S/σ , the latter ratio is known as the standardized capacity C. S/μ is a useful measure for practitioners because it represents the maximum number of years of water held in storage, while, for many theoretical studies, it is useful to standardize the capacity with respect to σ . Draft is also expressed as the ratio of mean annual inflow, $\alpha = D/\mu$, often as a percentage. Another parameter (m) that includes draft is known as the standardized net inflow [16] or drift [30,40] and is defined as follows:

$$m = \frac{1 - \alpha}{C_{\rm V}},\tag{1}$$

where $Cv = \sigma/\mu$ is the coefficient of variation of annual inflows. Hazen [15] was the first to adopt this parameter in his analysis of reservoir capacities for municipal water supplies. He denoted the parameter by the symbol 'k' but did not coin a term to describe it. This is a useful parameter which enables one to capture the impact of both streamflow variability, Cv, and reservoir yield (α) on reservoir storage.

The total capacities of individual reservoirs world-wide are listed in publications like ICOLD World Register of Dams [17]. However, typical values of capacity, expressed as S/μ or S/σ , and draft are not readily available. Nevertheless, there are several sources of data which have allowed us to build a picture of the variation of these reservoir parameters across three countries – Australia, South Africa and the United States. For example, the statistics based on 48 Australian reservoirs are as follows [23]:

- Of the 48 large reservoirs, the median value of drift is 0.66. Thirty-six reservoirs (75%) have a value of drift (m) < 1.0.
- Draft ratios vary between ~90% to less than 10% of the mean annual streamflow (MAF) into the reservoirs. The median value is 47%.
- For the same 48 reservoirs, reservoir capacities vary from more than $6 \times MAF$ to some being $<0.25 \times MAF$ or in terms of annual standard deviation from $>10 \times$ annual standard deviation (annual σ) to $<0.25 \times$ annual σ . The median size of the reservoirs is $1.28 \times MAF$ or in terms of standardized capacity $1.71 \times$ annual σ .

In South Africa, withdrawal rules are typically optimized in systems of interconnected reservoirs, so there is only a small subset of those with their own complete (not incremental) catchments:

- For 12 of the larger stand-alone reservoirs, the median drift is 0.63, of which one has a drift of 1.01, the remainder have drifts between 0.25 and 0.91.
- For these 12 reservoirs, draft ratios vary between 14% and 90% of MAF: the median value is 29%.
- For the same 12 South African reservoirs, capacities vary from more than $3.3 \times \text{MAF}$ to some being $< 0.7 \times \text{MAF}$ or in terms of annual standard deviation from $> 6.8 \times \text{annual } \sigma$ to $< 0.63 \times \text{annual } \sigma$. The median size of the reservoirs is $1.22 \times \text{MAF}$ or in terms of standardized capacity $1.20 \times \text{annual } \sigma$.

Graf [11] reviews the general characteristics of over 75 000 dams in the United States and Vogel et al. [46] evaluate the hydrologic characteristics of a smaller subset of just over 5000 of those dams. Dam behaviour differs quite

Table 1 Values of drift or standardized inflow, m, as a function of draft ratio α and annual Cv

Annual Cv	α (%)				
	25	50	75	90	100
0.1	7.5	5.0	2.5	1.0	0
0.5	1.5	1	0.5	0.2	0
1.0	0.75	0.5	0.25	0.1	0
2.0	0.375	0.25	0.125	0.05	0

Drift $(m) = (1 - \alpha)/Cv$.

Values indicated in italics denotes carry-over storage $(0 \le m \le Cv)$ [42].

dramatically in eastern and western regions of the US due to differences in hydrologic variability

- Boxplots of drift for thousands of reservoirs are presented in [46] illustrating that east of the Mississippi river drift is generally greater than 1 whereas it is generally less than 1 in western regions.
- Drafts range from around 40% to 95% of MAF in some eastern regions and are nearly uniformly distributed in some western regions.
- Storage capacities are generally less than the MAF in the eastern regions, but range from nearly zero to nearly 5xMAF in some western regions.

In summary, the large reservoirs in Australia and South Africa exhibit similar characteristics – typically, reservoir capacities are about 11/4 times mean annual flow, draft ratios vary from about 10% to 90%, and the median magnitude of drift is about 2/3. In US, large regional differences are observed with the eastern regions being characterized by higher drafts and smaller reservoir capacities than in the western regions.

Values of drift as a function of α and Cv are shown in Table 1 and, for all practical purposes, cover the range of values found globally. As a rough guide, reservoirs with m < Cv are considered to operate usually as over-year or carry-over storage reservoirs (these are reservoirs in which part of the stored water is carried over from one year and used in subsequent years), whereas those where $m \ge Cv$ are classified as within-year systems and would be usually expected to spill annually [46]. Vogel and Bolognese [41] had suggested $m \ge 1$ as the guide for within-year storage. As noted by the shading in Table 1 the two guidelines, $m \ge 1$ and $m \ge Cv$, are consistent. Montaseri and Adeloye [26] argue that other variables in addition to m and Cv, such as reliability, may be needed to classify reservoirs as carry-over or within-year storages. We emphasize that the classification between systems dominated by withinyear or over-year behaviour is more of a continuum, than a sharp distinction, given by any set of rules.

3. Reviewing some key approaches

This section compares five approaches (other than stochastic simulation) – Extended deficit analysis, behaviour analysis, sequent peak algorithm, Vogel and Stedinger empirical (lognormal) method and Phien empirical (Gamma) method – that are currently available for reservoir storage—yield analysis. We begin by describing each method along with its attributes and limitations.

3.1. Extended deficit analysis

The Extended Deficit Analysis (EDA) was proposed by Pegram [32] as a simple technique to compute the average recurrence interval of reservoir deficits (in other words, the mean recurrence interval between emptiness) directly from the historical inflows, excluding net evaporation and other losses. (Evaporation can be handled externally [23]).

The method is based on the storage equation applied to a semi-infinite reservoir (one that can spill but never empty) to determine the capacity S required to provide a chosen draft of given reliability:

$$Z_t = \min[0, Z_{t-1} + Q_t - D_t]$$
(for simultaneous inflows and drafts), (2)

$$Z_t = \min[0, Z_{t-1} + Q_t] - D_t$$
(for inflows and drafts out of phase), (3)

where Z_t and Z_{t-1} are storage values (≤ 0) at times t and t-1, (initial storage is assumed full: $Z_0=0$), Q_t is the inflow and D_t is the draft during the interval (t-1, t). The analysis computes the minimum storage between spills (i.e., when Z=0) as positive deficits:

$$Def_j = -\min[S_t \text{ between spills } j-1 \text{ and } j], j = 1, 2, \dots, r,$$
(4)

noting that the spill j = 0 occurs at t = 0 because $Z_0 = 0$, by definition. In applying the method here, the Pegram procedure was slightly modified by incorporating the lowest storage experienced between the last spill and the end of the record, although it is not technically a deficit by the definition of Eq. (4).

Because the deficits Def_j , $j=1,2,\ldots,r+1$ are separated by spills (except the r+1th deficit), they form a renewal process and are therefore mutually independent. Following Troutman [40], who showed that for a semi-infinite storage fed by inflows with $\alpha < 1$ (or m > 0) the maximum deficit asymptotically has an Extreme Value Type 1 (EV1) or Gumbel distribution, the "larger" deficits can therefore be considered to be EV1 distributed.

Once the series of r + 1 deficits is obtained, they are ranked from largest (i = 1) to smallest (i = r + 1) and for each deficit a sample average recurrence interval is calculated using Gringorten's plotting position [12]:

$$T = \frac{N + 0.12}{i - 0.44},\tag{5}$$

where N is the number of years in the historical record and i is the rank of the deficit. For each value of T, the EV1 reduced variate, y, is calculated from

$$y = -\ln\left[-\ln\left(1 - \frac{1}{T}\right)\right]. \tag{6}$$

An example of the application of EDA is shown in Fig. 1 for the Mazoe River at Dam (Zimbabwe). From a practical point of view, a suggested lower cutoff, to remove small nuisance deficits, is set at a recurrence interval T of approximately 8 years, corresponding to y=2. From Fig. 1, the deficit for T=100 years calculated from the fitted trend line (straight on a Gumbel plot) is 78×10^6 m³, which is the size of reservoir required to provide a given draft (75% of the mean annual flow was adopted for this case) with a reliability of supply of 99%. In the analysis reported later we have estimated the reservoir deficits for average recurrence intervals of 20 and N years when two or more data points are available, where N is the length of record.

3.1.1. Attributes and limitations

Although EDA is based on the analysis of net inputs to a semi-infinite reservoir, it does not suffer from the inadequacy of defining reliability, as occurs with other semi-infinite reservoir techniques [24] because the deficits are independent events rather than a sequence of dependent storage values formed from a sequence of flows into a semi-infinite reservoir. Because it is nonparametric and exploits the record in its entirety, it implicitly incorporates the characteristics of the record – mean, variability, serial dependence (in all its complexity) without having to extract statistics. The method has the advantage that it yields a direct estimate of the storage needed to supply a given demand with a specified reliability. The method of determination of the recurrence interval of failure is to relate the deficits Def, and their individual mean recurrence times, T, by regression of Def, onto the EV1 reduced variate, but excluding those deficits with $T \le 8$ years. Because this is a form of censoring (rather like using the 'Peaks Over Threshold' series instead of annual floods), the computed recurrence times will differ by a small amount from the usual annual interpretation of recurrence interval of failure to supply. This difference (about 0.5 years for T > 8 years) can be ignored for all practical purposes.

3.2. Behaviour analysis

In behaviour or simulation analysis, the changes in storage content of a finite reservoir (one that can spill and empty) are computed using the water balance equation

$$Z_t = Z_{t-1} + Q_t - D_t - \Delta E_t - L_t, \tag{7}$$

subject to $0 \le Z_t \le S$ where Z_t is the storage content at time t (it starts empty at $Z_0 = 0$), and the remaining terms are fluxes during the interval (t-1, t), Q_t is the flow into the reservoir, D_t is the controlled release, ΔE_t is the net evaporation loss from the reservoir, L_t represents other losses, and S is the active reservoir capacity. For the analysis of the hypothetical storages that follows, we assume there is no net evaporation or other losses from the reservoir.

To estimate the required reservoir capacity S, one simulates the state of the storage based on Eq. (7) using historical data or stochastic sequences assuming the reservoir is initially full. In reality this may not be so as reservoirs are often brought into operation before the reservoir has first spilled. The appropriate size S is typically one that will provide the target draft D at some level of reliability or performance. There are several measures of reliability and performance available [23], however, we use time reliability at either an annual or monthly level. Essentially this is the ratio (or percentage) of the number of time units the reservoir was able to meet the target demand divided by the total number of time units in the simulation. Time reliability adopted in

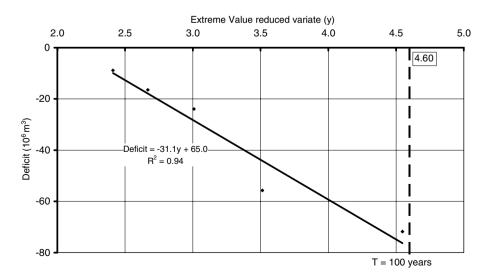


Fig. 1. Example of Extended Deficit Analysis applied to the Mazoe River at Dam (Zimbabwe) for a constant draft of 75% using annual data. The figure shows the relationship between reservoir storage deficit and extreme value type 1 reduced variate.

behaviour analysis and most often in practice is different from the reliability statements adopted by the other methods considered in this study.

The yield estimate from this calculation is known as *steady state* yield in contrast to the *firm* yield estimated by the sequent peak algorithm which is discussed next. Because the historical record length is usually too short to estimate steady state conditions (see the discussion that follows), McMahon and Adeloye [23] have introduced the term *pseudo-steady state* to describe behaviour analysis yield estimates based on historical data lengths.

3.2.1. Attributes and limitations

A behaviour (or simulation) analysis is a simple and visual procedure to estimate storage capacity and is not restricted by the characteristics of the inflows. Unlike some of the analytical approaches, evaporation and operating rules that are a function of reservoir storage levels can be easily taken into account [23].

Depending on the length of the annual inflow data, storage size for high reliabilities cannot be estimated. For example, with 50 years of data, the required storage size for 99% annual time reliability (1/100 years probability of failure) cannot be estimated.

Based on stochastically generated annual streamflows, Pretto et al. [35] found that biases occur in the mean and higher order quantiles of storage estimates before the estimated storage size converges to a stationary value after a long sequence (typically 1000 years or more). Adeloye et al. [2] noted that by restricting the shortfall during failures the biases largely disappear.

3.3. Sequent peak algorithm

A number of variants of the sequent peak algorithm (SPA) are available that accommodate storage dependent losses. In this paper, we restrict our analysis to the application of the basic SPA to a single reservoir. Assuming the initial storage in a semi-infinite reservoir is zero (in other words, the reservoir starts full as in EDA), we apply the water balance Eq. (2) for all years or months in the streamflow record of length *N*:

$$Z_t = \min[0, Z_{t-1} + Q_t - D_t], \tag{8}$$

where Z_t is the storage (≤ 0) at time t (again $Z_0 = 0$), and D_t and Q_t are the draft and the inflow during the interval (t-1, t). If $Z_N \neq Z_0$ we continue with Eq. (8) for the concatenated inflow sequence. The required active reservoir capacity is given by:

$$S = -\min(Z_t) \text{ over all } t = 1, 2, \dots, N(\text{or } 2N \text{ if } Z_N \neq Z_0).$$

$$(9)$$

3.3.1. Attributes and limitations

Using the historical inflow data, SPA computes the storage required to provide the firm yield, which is the yield that can be met over a particular planning period with

no failure. This approach has been widely used in the United States and elsewhere. Furthermore, the design capacity of many reservoirs world-wide has been determined using either the Rippl [36] graphical method or the SPA which is a numerical version of that technique. Borrowing from EDA, the steady-state reliability associated with S is roughly 1-1/N, though since only one failure is allowed over the N-year period, this is a very poor estimate of steady-state reliability.

As SPA is equivalent to the Rippl graphical mass curve procedure [36], it suffers from the same limitations. First, the estimated storage is based on the critical historic low flow sequence and says little about the reliability (expressed as a probability) of meeting the target draft. Second, fluxes (including evaporation) dependent on storage content cannot be taken into account in the simple SPA procedure. However, Lele [20] and Adeloye and Montaseri [1] offer more complex algorithms to overcome this inadequacy.

3.4. Vogel and Stedinger empirical procedure

Vogel and Stedinger (V–S) [44] showed that for a reservoir system fed by AR(1) lognormal streamflows, the standardized storage C (capacity divided by the standard deviation) for a failure-free operation (SPA approach) is a random variable described by a three-parameter Lognormal distribution. The form of the V–S relationship is:

$$S_p = \sigma[\vartheta_s + \exp(\mu_\ell + z_p \sigma_\ell)], \tag{10}$$

where S_p is the pth quantile of the distribution of required reservoir capacity for 100% failure-free operation over a specified planning period N, z_p is the standardized Normal variate at p%, σ is the standard deviation of annual streamflows, μ_ℓ and σ_ℓ are mean and standard deviation of the logarithms of the storages defined in Eqs. (11) and (12), and ϑ_s is the lower-bound of the storage. The moments μ_ℓ and σ_ℓ are computed as follows:

$$\mu_{\ell} = \ln \left[(\mu_{\rm s} - \vartheta_{\rm s}) \left(1 + \frac{\sigma_{\rm s}^2}{(\mu_{\rm s} - \vartheta_{\rm s})^2} \right)^{-0.5} \right],\tag{11}$$

$$\sigma_{\ell}^2 = \ln \left[1 + \frac{\sigma_{\rm s}^2}{\left(\mu_{\rm s} - \vartheta_{\rm s}\right)^2} \right],\tag{12}$$

where μ_s and σ_s are the mean (expected value) and standard deviation of the untransformed (real space) storage capacity S.

Vogel and Stedinger [44] carried out extensive Monte Carlo simulations (streamflows were based on an AR(1) log-normal model) within the ranges $0.2 \leqslant \text{Cv} \leqslant 0.5$, $0.1 \leqslant m \leqslant 1$, $0 \leqslant \rho \leqslant 0.5$ and $20 \leqslant N$ years $\leqslant 100$, resulting in the following equations to compute the parameters in Eqs. (10)–(12).

$$\mu'_{s} = \exp(a + bm)\alpha^{c} m^{m(d\rho + eN)} N^{(f+g\ln(m))} \left(\frac{1 + \rho}{1 - \rho}\right)^{h\ln[N]}, \quad (13)$$

$$\sigma_{\rm s}^{\prime 2} = \exp\left[a + b\alpha + \frac{cN}{m} + \left(\frac{d}{N} + \frac{e}{m}\right) \left(\frac{1+\rho}{1-\rho}\right)\right] N^{f \ln[m]} \left(\frac{1+\rho}{1-\rho}\right)^{g \ln[N]},\tag{14}$$

$$\vartheta_{s}' = a\rho + \left(bN + \frac{c(1+\rho)}{(1-\rho)}\right) \ln[m] + N\left(d + \frac{e}{m} + fm \ln[N] + g \ln\left(\frac{1+\rho}{1-\rho}\right)\right), \tag{15}$$

where a through h are the empirical parameters based on simulation, α is the target draft ratio D/μ , and ρ is the lag-one serial correlation of annual streamflows. For each equation a separate set of parameters is required, giving a total of 22; they are listed in [23,44]. Analogous regressions are given for the case of AR(1) normal inflows in [41].

3.4.1. Attributes and limitations

The Vogel and Stedinger empirical procedure estimates the expected value and variance of SPA reservoir storage capacity assuming both the inflows to the reservoir and the standardized storages are lognormally distributed. Although the method is based on six equations and 22 parameters, its application is straightforward.

Being an empirical procedure, it should really only be applied within the range of values of m, Cv, N and ρ that were used to define the 22 parameters. However, as noted in Section 5.4.2, the results of applying the technique to the set of global rivers suggests it can be used across the full range of rivers with caution.

Evaporation is not explicitly taken into account but can be handled externally (see for example McMahon and Adeloye [23]).

3.5. Phien empirical procedure

Phien [33] also used the SPA procedure to explore the distribution of the required reservoir capacities but adopted the Gamma rather than lognormal distribution to define stochastically generated reservoir inflows. Phien's criteria were based on drift, lag-one serial correlation and record-length as follows: $0 \le m \le 0.50$, $0 \le \rho \le 0.5$ and $20 \le N$ years ≤ 50 . This is a more limited data set than adopted by Vogel and Stedinger [44]. Based on the simulated results, Phien developed several empirical relationships, far simpler in form than the empirical models in [41,44]; his equations for the expected value and the standard deviation of standardized storages are:

$$\mu_{\rm S} = 1.467 N^{0.466} \left[\frac{1+\rho}{1-\rho} \right]^{0.531} \left[\frac{1-m}{1+m} \right]^{2.047},\tag{16}$$

$$\sigma_{\rm S} = 1.787 N^{0.243} \left[\frac{1+\rho}{1-\rho} \right]^{0.855} \left[\frac{1-m}{1+m} \right]^{2.198}, \tag{17}$$

where μ_S and σ_S are the mean (expected value) and standard deviation of the storage values.

3.5.1. Attributes and limitations

In contrast to the V–S procedure, Phien's method assumes the annual streamflows are Gamma distributed and uses only two simple equations to compute the expected value and the variance of standardized reservoir capacities. However, the range of the characteristics of streamflows is more limited than those used in the V–S method. Furthermore, as noted in Section 5.4.2, the procedure should not be used outside the range specified.

Following Section 4, in which the global set of streamflow records are described, the above methods for estimating required reservoir capacities are applied to the global data.

4. Streamflow data

The global streamflow data set used in this analysis consists of continuous monthly time series with 25 or more vears of historical streamflows for 729 unregulated rivers. which is a sub-set of 1221 rivers with 10 or more years of data. The locations of the 729 rivers, shown in Fig. 2, suggest that the data are reasonably well distributed worldwide. The larger data set was initially collated by the first author in the eighties [25] with subsequent additions and revisions [28,29]. We believe the streamflow records are not significantly regulated by reservoirs nor affected by major diversions upstream of the gauging stations. Fig. 3 shows a plot of catchment area against record length. The catchment areas vary from 134 to 61300 km² (10th to 90th percentile) and have a median of 1370 km²; historical record lengths are from 27 to 66 years (10th to 90th percentile) with a median of 38 years.

5. Key comparisons

5.1. Some basic flow characteristic of the global data set

The flow characteristics of the global data set exhibit some features that should enable us to evaluate the adequacy of several of the S-Y techniques discussed above. First we explore the probability distribution of annual streamflows in Fig. 4 using an L-moment diagram. Vogel and Fennessey [42] showed that L-moment diagrams are always preferred over ordinary product moment diagrams for evaluating goodness of fit of alternative distributions. They showed that conclusions derived from ordinary moment diagrams can be very misleading. Fig. 4 compares the sample L-Cv versus the L-skewness of the 729 annual streamflows with theoretical relations for the lognormal and Gamma distributions. In Fig. 4 the data points compared with the theoretical curves suggest that the Gamma distribution provides a slightly more satisfactory representation of the distribution of annual streamflows than the lognormal distribution. This is justified by Vogel and Wilson [45] who found that the Gamma distribution provided an excellent fit to 1481 annual flow series in the US and was preferred over the lognormal distribution. Additional

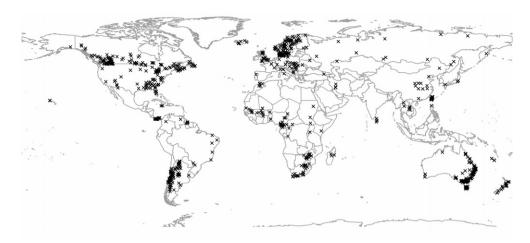


Fig. 2. Location of 729 stream gauging stations with 25 years or more of continuous monthly and annual streamflow data selected from the global data set

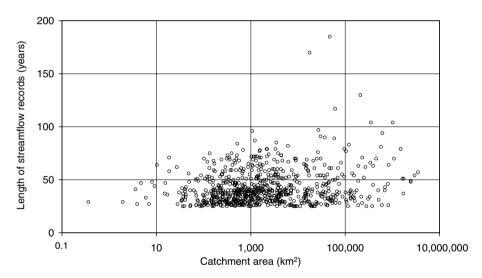


Fig. 3. Length of available continuous streamflow data versus catchment area for the selected 729 streamflow records.

unpublished analysis by the authors of the larger global data set (1221 rivers) noted in Section 4 indicates that the Gamma probability density function (pdf) fits the annual data slightly better than the lognormal pdf. Neither distribution can exhibit negative skewness, in spite of the fact that some of the sample estimates of L-skewness are negative.

Of the 729 rivers, 141 (19.3%) exhibit statistically significant positive auto-correlations. The median value of these correlations is 0.44, 12 rivers having significant negative values. This issue is discussed in some detail in a companion paper yet to be published.

5.2. Extended deficit analysis

In our analysis, EDA was applied to annual streamflow data. As explained in Section 3.1, EDA analysis determines for each streamflow record a number of independent deficits, the number depending on the length of record and

the particular sequence of flows that are being analyzed. Fig. 5 shows for the 729 rivers in the global data set $(N \ge 25)$, the number of independent deficits for $\alpha = 0.75$ compared with record length.

Fig. 5 illustrates there is a weak yet positive relationship between the number of deficits and minimum record length. It turns out, for this choice of parameters, that the upper bound line is very well defined by the equation $N = 10 \ y - 4$, where y is the number of deficits, so that for y = 2 (the smallest number of deficits to allow an extrapolation of deficit as a function of T) the minimum record length is 16. The spread of points to the right of such a line indicates that some long records have very few deficits; typically these have small Cvs. For example, for rivers with 50 or more years of data that have only two deficits, the median annual Cv is 0.16. An appreciable number of records have no deficits at all, so that the reservoir spills continuously; these records also have small Cvs. In the analysis we limited the regression extrapolation (see

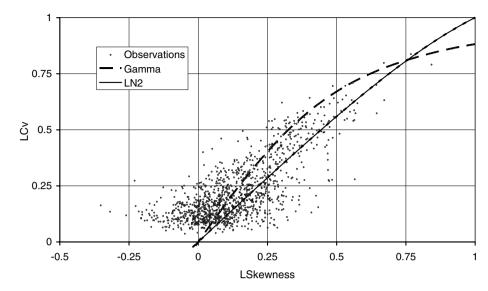


Fig. 4. Sample estimates of L-Cv versus L-skewness measured from the 729 sequences of annual streamflow.

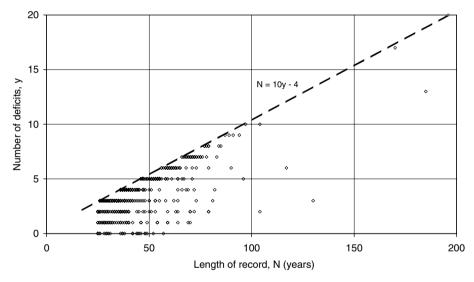


Fig. 5. Comparison for each river of the number of independent deficits based on Extended Deficit Analysis (1 in 100 years recurrence interval) for $\alpha = 0.75$ versus the length of the historical record (years) for all 729 records.

Fig. 1) to rivers with two or more deficits – thus 83% of the rivers in the global set remained for analysis. Four or more deficits were observed for 42% of the rivers, mainly those with longer records.

Comparison among EDA, behaviour and SPA storage estimates are presented in Figs. 6 and 7 and discussed in the next two sections.

5.3. Behaviour analysis

Behaviour (or simulation) analysis is another procedure (like EDA) that estimates reservoir yield, corresponding to a given capacity and time-based reliability. However, it is a more accurate and flexible procedure than EDA because net evaporation and other storage dependent processes, like restricted releases, can be easily taken into account

and it is more flexible than SPA because steady-state reliability estimates are available. In addition, because it is applied to finite reservoirs, by trial one can compute from the output, the three conditions of emptiness as defined by Pegram [31], namely mean recurrence time of emptiness, mean first passage time from full to empty and mean first passage time from empty to full. Behaviour analysis is also used to determine the volumetric reliability defined as the ratio of the total volume of water supplied to the volume demanded. It has also been used to estimate system reliabilities using stochastically generated flow sequence rather than using only the historical record.

The maximum time reliability that can be estimated using behaviour analysis is restricted to the number of time units available for simulation. For example, using annual flows a minimum of 20 years of data is needed to compute

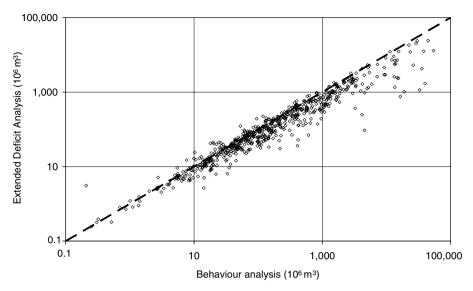


Fig. 6. Relationship of reservoir capacity estimates based on annual streamflows and for $\alpha = 0.75$ between extended deficit analysis for 1/20 year recurrence interval of failure and behaviour estimate for 95% time reliability. The 1:1 line is dashed.

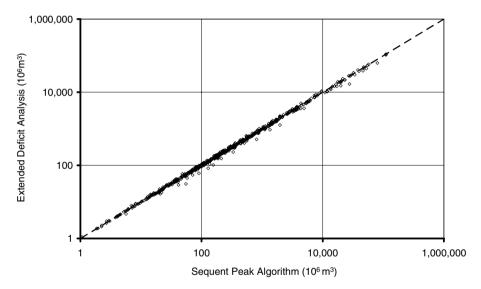


Fig. 7. Comparison between extended deficit analysis reservoir capacity estimates (based on Gringorten formula adjusted using N years average recurrence interval of failure) and sequent peak algorithm estimates for $\alpha = 0.75$. The 1:1 line is dashed.

the storage size to meet a 95% annual reliability. This can be achieved for all rivers in the data set (as we have set a minimum length of 25 years). However, to achieve a 98% reliability would require a minimum of 50 years of data and this could be achieved for only 27% of the rivers studied.

5.3.1. EDA compared with behaviour

Fig. 6 compares EDA reservoir capacities based on annual flows with those estimated by a behaviour analysis for 75% target draft and for a probability of failure of 1:20 years (EDA) and 95% annual time reliability (behaviour). One observes there are considerable differences in the reservoir capacity estimated by the two techniques. A major reason for this is the difference in definitions of failure. In our

analysis of a behaviour simulation we have adopted the failure criterion used in water engineering practice, namely, the proportion of time units the reservoir fails to meet the target demand, whereas for EDA the criterion is the mean number of time units between failures. Consequently, for each EDA failure (emptiness), there will be a corresponding period of failure observed during the equivalent Behaviour analysis. However, for the latter each failure may last longer than one time unit. As a result, for the same reliability criterion, say 95% (or 5% failure), there will be more time units of failure associated with the behaviour simulation than for EDA. This means the reservoir capacity estimated using the behaviour method will be larger than the EDA estimate for the same reliability or failure condition. This is observed in Fig. 6 with most points lying to the right

of the 1:1 line. For example, for 75% target draft, the required reservoir capacity estimated by behaviour compared with EDA varies from being 28% larger for a $10 \times 10^6 \, \text{m}^3$ hypothetical reservoir to 86% larger for a $10000 \times 10^6 \, \text{m}^3$ hypothetical reservoir.

5.4. Sequent peak algorithm

The SPA is explored in some depth as it is used widely throughout the world to estimate S-Y relationships. In the SPA analysis that follows, $\alpha = 0.75$ has been adopted.

5.4.1. SPA compared with EDA

Fig. 7 compares the storage estimates based on an SPA analysis versus EDA estimates where, for the latter, the recurrence interval to specify the deficit is based on an adjustment of the record length N using the Gringorten formula (Eq. (5)). Despite the fact that the techniques have different theoretical bases, the storage estimates of the two procedures for the 599 rivers are virtually identical as confirmed by the exceptionally high correlation (Spearman Rank Correlation = 0.999).

5.4.2. Empirical and computed estimates of SPA

To provide further insight into the SPA approach, we have compared the storage estimates by the Vogel and Stedinger [44] and the Phien [33] empirical models outlined in Sections 3.4 and 3.5 (both of which estimate the SPA-based storage for a given firm yield) with the SPA values determined from annual flows in the global data set.

Initially, the Vogel and Stedinger (V–S) estimates were compared with the SPA estimates for the rivers that fall within the limitations: $0.1 \le m \le 1.0$, $20 \le N \le 100$, $0 \le \rho \le 0.5$ and $0.1 \le \text{Cv} \le 0.5$ [44]. Of the 729 rivers in the global data set, 221 (30%) fulfilled these conditions. In applying the V–S method we used Eq. (13) which allows

one to compute the expected value of the standardized storage (defined as the estimated mean capacity divided by the standard deviation of annual inflows).

The results of the comparison are shown in Fig. 8 which can be considered as an independent test of the V-S model. Although analysis of the regression equation in log space indicates that the slope (0.977) is significantly different from one, the figure does suggest an overall satisfactory fit. Compared with reservoir capacities estimated using the historical data, the V-S model underestimates by about 3% and 16% when using volumes of 10×10^6 m³ and 1000×10^6 m³, respectively, for comparison. It would be difficult to confirm whether the minor differences observed in Fig. 8 are due to the fact that we are comparing a single SPA estimate based on historical flows to an expected value given by Eq. (13) or due to some other cause, for example, the fact that the annual flows may be better described by a Gamma distribution than a lognormal distribution which is the assumption in the V-S model. An analysis to identify this small bias is outside the scope of this paper.

Given the satisfactory fit of the V–S model in Fig. 8, we have plotted in Fig. 9 the storage estimates based on the V–S model compared with the SPA estimates using the entire global data set. The slope of the regression equation in log space (without intercept) is 0.969 compared with 0.977 in Fig. 8. Given that 70% of the rivers are outside the parameter ranges used to develop the coefficients for the model, the figure suggests the model can be used with due caution over the full range of flow, data length and drift values. However, it should be noted that, for large storages, the V–S method underestimates considerably. For example, for an SPA storage of $10\,000\times10^6$ m³, the trend-line suggests that V–S underestimates storage by 32%.

In Fig. 10, we compare the Phien model (Eq. (16)) for $\alpha = 0.75$ with SPA estimates for rivers from the global data

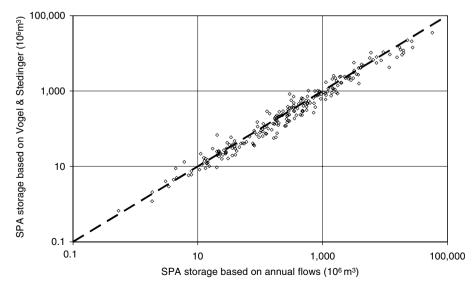


Fig. 8. Expectation of reservoir capacity estimates based on empirical Vogel and Stedinger Eq. [44] compared with individual sequent peak algorithm estimates computed from annual data for $\alpha = 0.75$. (Applicable range $0.1 \le m \le 1.0$, $0 \le \rho \le 0.5$, $20 \le N \le 100$ and $0.1 \le Cv \le 0.5$) The 1:1 line is dashed.

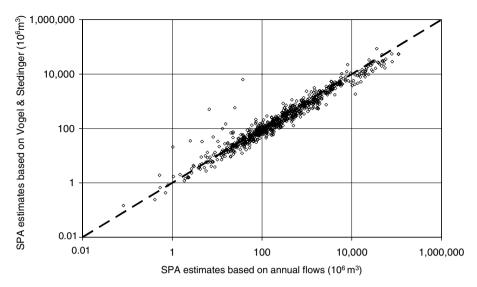


Fig. 9. Comparison between Vogel and Stedinger empirical equation estimates of reservoir capacity compared with sequent peak algorithm estimates computed from the global data set of annual flows for $\alpha = 0.75$. The 1:1 line is dashed.

set that cover the following range of conditions for which the Phien empirical equation was developed: $0 \le m \le 0.50$, $0 \le \rho \le 0.5$ and $20 \le N$ years ≤ 50 . Only 19% of the 729 global rivers meet these conditions. Overall, the model underestimates the storages by about 25%, and the regression slope (0.952) is significantly different from one at the 5% level of significance. When applied to all of the global rivers, the method severely underestimates the reservoir capacities, on average by about 80%, and we therefore recommend it not be applied outside the range of parameters upon which it is based.

6. Storage estimates based on monthly and annual data

The discussion to date has focused on adopting annual streamflow to estimate reservoir capacities. In the case

where within-year storage is an important component of reservoir capacity, monthly data should be used or an appropriate adjustment made to the capacities estimated using annual streamflows. To explore how significant these differences might be, we compare for the global data set in Fig. 11 the reservoir capacities computed using annual data $(S_{\rm ann})$ with those estimates from monthly data $(S_{\rm mon})$ using SPA analysis. In the figure the ratio of the SPA reservoir capacity computed from monthly flows to that based on annual flows is plotted against drift. The figure also shows separately $S_{\rm mon}/S_{\rm ann}$ values for $m > {\rm Cv}$ from $m < {\rm Cv}$. As expected the majority of values of $S_{\rm mon}/S_{\rm ann}$ for $m < {\rm Cv}$ fall in the range of the monthly to annual storage ratios of 1–1.5 with corresponding drift m values between 0 and 0.75.

The results show that for 87% of the rivers in the global data set, the estimated storage requirements for α ranging

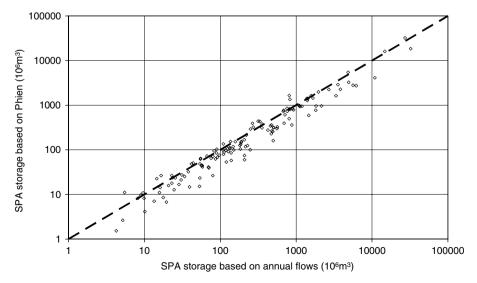


Fig. 10. Reservoir capacity estimates based on empirical Phien equation [33] compared with sequent peak algorithm estimates computed from annual data for $\alpha = 0.75$. (Applicable range $0 \le m \le 0.5$, $0 \le \rho \le 0.5$ and $20 \le N \le 50$). The 1:1 line is dashed.

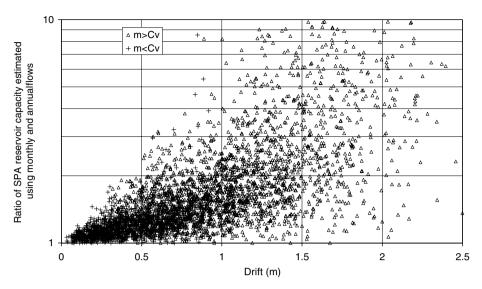


Fig. 11. Scatter plot of the ratio between storage estimates based on monthly and annual data (using the sequent peak algorithm estimates for $\alpha = 0.1$ –0.9 occurring simultaneously with inflows) against drift.

from 0.1 to 0.9 using annual data needs to be increased by 10% or more to account for the within-year variation and for 44% of the rivers the estimated storage needs to be increased by 50%. Thus, it is concluded that annual flow data alone is an inadequate measure of required storage size for many streams and this is particularly true for regions with high streamflow variability (Cv) or drift values in excess of unity. These conclusions are based on the use of Eq. (2) which ignores seasonality effects. Using Eq. (3) would bracket the monthly behaviour from above because is assumes no overlap between inflow and draft – the extreme of an instantaneous input or draft [30].

Adeloye et al. [3] developed an empirical equation to adjust annual SPA storage values to give total (over-

year + within-year) storage capacities. The equation was based on 15 rivers (12 were used for calibration and 3 for validation) that represented the world-wide range of annual streamflow characteristics. The equation is as follows:

$$S_T = -0.222 + 0.322\text{Cv} + 0.6\alpha + 1.025S_A, \tag{18}$$

where S_T is the total SPA storage estimate and S_A is the over-year SPA storage estimate based on annual flow analysis. Cvs ranged from 0.19 to 1.07 and α from 0.4 to 0.8.

In Fig. 12 we compare for the 729 global rivers considered here, the SPA storage estimates based on Eq. (18) for $\alpha = 0.75$, with those computed using the monthly data. The comparison is very encouraging for $\alpha = 0.75$, although the regression slope (1.024) in log space is significantly different

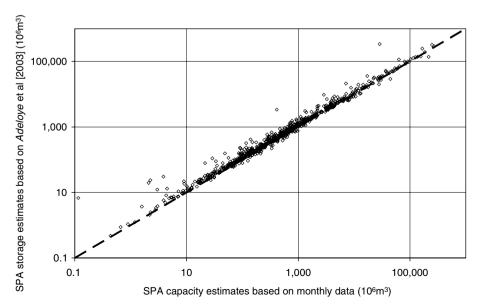


Fig. 12. Comparison of sequent peak algorithm reservoir capacity estimates based on Adeloye et al. empirical Eq. [3] compared with estimates based on monthly streamflow data for $\alpha = 0.75$. The 1:1 line is dashed.

from one. For reservoir capacities of 10×10^6 , 1000×10^6 and $100\,000 \times 10^6$ m³, Eq. (18) overestimates the total storage using monthly data by 37%, 19% and 4%, respectively. Because of the bias in the equation that is represented by these results, analysts need to use care in using Eq. (18) to estimate the total capacity of smaller storages.

7. Summary comments

A review of the literature suggests this is the first paper in which several reservoir capacity—yield techniques have been compared using a large number of representative rivers from a global data set. In the past, comparisons have been based on data for only a few rivers or for rivers from a restricted geographical area. The five techniques examined herein represent the two general approaches adopted for examining the relationship between reservoir capacity and yield. The two approaches [43] are the no-failure firm yield approach, which is the yield that can be met over a particular planning period with no failure, represented by the sequent peak algorithm (and the Vogel and Stedinger and Phien empirical methods) and the steady state yield approaches represented by the Extended Deficit Analysis and behaviour analysis.

Before we completed these analyses the authors had little idea of the level of variation in reservoir capacity estimates one could expect between the various techniques when applied to actual river flow data that cover the global range of annual streamflow characteristics. To determine standard errors of estimation in regression analyses like those in Figs. 7, 8 and 10, a basic assumption is that the regression residuals are normally distributed. Of the three figures, this assumption is violated only for Fig. 7, but we believe the degree of non-normality in conjunction with the large sample size of 599 values in this case provides sufficient confidence [4, p. 109] to carryout the following comparison. Based on the regressions between the two log axes in Figs. 7, 8 and 10 without intercept terms, the following standard errors of estimates can be computed in log space using $\sqrt{\exp(s^2)-1}$ [13], where s is the standard deviation of the model residuals when the model is fit in log space, as follows:

Fig. 7 (EDA versus SPA) $\pm 8.7\%$ Fig. 8 (V&S SPA versus SPA from restricted data) $\pm 39.4\%$ Fig. 10 (Phien SPA versusu SPA from restricted data) $\pm 43.3\%$

These results suggest that when either of the empirical methods V&S or Phien are used, one should be able to achieve standard errors of estimate less than about $\pm 40\%$.

In water resources system planning, empirical equations like Eqs. (13) and (16) are often used for initial hydrologic assessment of a range of potential reservoir sites. Although these errors appear large (this is the first time values have been computed for a large data set), their magnitudes are

not inconsistent with, for example, errors in mean flow estimates for highly variable rivers.

8. Conclusions

A number of conclusions follow from this assessment of the application of five reservoir storage—yield techniques — extended deficit analysis, behaviour analysis, sequent peak algorithm, Vogel and Stedinger empirical method and Phien empirical method — to estimate the capacity of hypothetical reservoirs located on 729 rivers distributed globally and also from previous research results. The rivers with at least 25 years of continuous monthly streamflow data cover the range of statistical characteristics observed world-wide. The following conclusions have been reached:

- 1. Seventy five percent of large Australian reservoirs and 91% of large South African reservoirs have values of drift m < 1. In the eastern US roughly 50% of the dams have drift <1 whereas in the western US approximately 75% of the dams have drift <1.
- For the same set of Australian and South African reservoirs, the median value of draft ratio is approximately 47% and 29% respectively, and median storage size is 1.28 and 1.22 × MAF respectively or, in terms of standardized capacity, 1.71 and 1.20 × annual σ, respectively.
- 3. Unlike other procedures based on semi-infinite reservoirs fed by historical flows, Extended Deficit Analysis (EDA) provides a simple, but theoretically rigorous, estimate of storage size in terms of recurrence interval of failure for a specified draft.
- 4. Ninteen percent of the rivers in the global data set have statistically significant positive lag-one auto-correlations for which the median estimated value is 0.44.
- 5. We found the Extended Deficit Analysis to be a useful procedure to estimate storage for a given draft ratio α . However, because we restricted the analyses to rivers yielding at least two deficits, which the method requires, the technique was limited to 83% of the global rivers.
- 6. Behaviour analysis was found to be a suitable procedure to estimate reservoir capacity. However, at the annual time step it is restricted by the length of data. For example, to estimate a capacity for 98% annual time reliability, at least 50 years of data are required which is available for only 27% of the global rivers considered here.
- 7. Overall, the Vogel and Stedinger empirical equations [44] led to a satisfactory agreement when compared to SPA estimates based on historical streamflows. Using data within the range for which Vogel and Stedinger empirical equations were developed, the procedure under-estimates the SPA capacities by between 3% and 16% compared with those estimated using the historical streamflows. When applied to all the global rivers (many outside the range for which the model was developed), the storage estimates were satisfactory, except that large storages may be underestimated by up to 32%.

- 8. The Phien model [33] underestimates the storages by about 25% within its specified range. The model performed poorly when applied to the whole data set, underestimating storages by up to 80%.
- 9. We note that, as a general rule, storages computed using annual data severely underestimate the storage capacity computed when using monthly streamflows when m > Cv and only moderately when m < Cv. If monthly data are not available, a total storage capacity can be obtained using the Adeloye et al. [3] empirical equation which provides a reasonable correction to obtain combined within-year and over-year storage needs.

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References

- [1] Adeloye AJ, Montaseri M. Adaption of a single reservoir technique for multiple reservoir storage-yield-reliability analysis. In: Zebidi H, editor. Water: a looming crisis? Proc of Int Conf on World Water Resources at the Beginning of the 21st Century, UNESCO, Paris, 1998. p. 349-55.
- [2] Adeloye AJ, Montaseri M, Garmann C. Curing the misbehavior of reservoir capacity statistics by controlling shortfall during failures using the modified sequent peak algorithm. Water Resour Res 2001;37(1):73–82.
- [3] Adeloye AJ, Lallemand F, McMahon TA. Regression models for within-year capacity adjustment in reservoir planning. Hydrol Sci J 2003;48:539–52.
- [4] Afifi AA, Clark V. Computer-aided multi variate analysis. Chapman & Hall/CRC; 1999.
- [5] Alexander GN. The use of the Gamma Distribution in estimating regulated outputs from storages. Civil Eng Trans, Inst Eng, Australia 1962;CE4(1):29–34.
- [6] Buchberger SG, Maidment DR. Diffusion approximation for equilibrium distribution of reservoir storage. Water Resour Res 1989;25(7):1643–52.
- [7] Gani J. Some problems in the theory of provisioning and of dams. Biometrica 1955;42:179–200.
- [8] Ghosal A. Emptiness in the finite dams. Ann Math Statist 1960;31:803–8.
- [9] Gould BW. Statistical methods for estimating the design capacity of dams. J Inst Eng, Australia 1961;33(12):405–16.

- [10] Gould BW. Discussion of Alexander GN. Effect of variability of stream-flow on optimum storage capacity. In: Proc of Water Resources Use and Management of a symp held in Canberra. Melbourne; Melbourne University Press; 1964. p. 161–4.
- [11] Graf WL. Dam nation: A geographic census of American dams and their large-scale hydrologic impacts. Water Resour Res 1999;35(4): 1305–12.
- [12] Gringorten II. A plotting rule for extreme probability paper. J Geophys Res 1963;68(3):813-4.
- [13] Hardison CH. Prediction error of regression estimates of streamflow characteristics at ungaged sites. US Geological Survey Prof. Paper 750-C, US Government Printing Office, Washington, DC, 1971. C228-C236.
- [14] Hashimoto T, Stedinger JR, Loucks DP. Reliability, resiliency and vulnerability criteria for water resource system performance evaluation. Water Resour Res 1982;18(1):14–20.
- [15] Hazen A. Storage to be provided in impounding reservoirs for municipal water supply. Trans Am Soc Civil Eng 1914;77: 1539–640.
- [16] Hurst HE. Long term storage capacity of reservoirs. Trans Am Soc Civil Eng 1951;116:770–99.
- [17] ICOLD. World Register of Dams 2003, International Commission on Large Dams, 2003; Paris, France.
- [18] Klemes V. Common Sense and Other Heresies: Selected Papers on Hydrology and Water Resources. Ont., Canada: Canadian Water Resources Association; 2000.
- [19] Langbein WB. Queuing theory and water storage. J Hydraul Division, Am Soc Civil Eng 1958;84(HY5):1–24.
- [20] Lele SM. Improved algorithms for reservoir capacity calculation incorporating storage – dependent and reliability norms. Water Resour Res 1987;23(10):1819–23.
- [21] Lloyd EH. A Probability Theory of Storage with Serially Correlated Inputs. J Hydrol 1963;1:99–128.
- [22] Loucks DP. Quantifying trends in system sustainability. Hydrol Sci J 1997;42(4):513–30.
- [23] McMahon TA, Adeloye AJ. Water Resources Yield. Colorado: Water Resources Publications, LLC; 2005.
- [24] McMahon TA, Mein RG. Reservoir Capacity and Yield. Amsterdam: Elsevier Scientific Publishing Company; 1978.
- [25] McMahon TA, Finlayson BL, Haines A, Srikanthan R. Global runoff – continental comparisons of annual flows and peak discharges. Cremlingen_Destedt, Germany, Catena, 1992.
- [26] Montaseri M, Adeloye AJ. Critical period of reservoir systems for planning purposes. J Hydrol 1999;224:115–36.
- [27] Moran PAP. The theory of storage. London: Methuen; 1959.
- [28] Peel MC, McMahon, Finlayson BL. Continental differences in the variability of annual runoff – update and reassessment. J Hydrol 2004;295:185–97.
- [29] Peel MC, McMahon TA, Finlayson BL, Watson FGR. Identification and explanation of continental differences in the variability of annual runoff. J Hydrol 2001;250:224–40.
- [30] Pegram GGS. Recurrence times of draft patterns from reservoirs. J Appl Probab 1975;12(3):647–52.
- [31] Pegram GGS. On reservoir reliability. J Hydrol 1980;47:269-96.
- [32] Pegram GGS. Extended deficit analysis of Bloemhof and Vaal Dam inflows during the period (1920–1994). Report to Department of Water Affairs and Forestry, Pretoria, South Africa, 2000.
- [33] Phien HN. Reservoir storage capacity with gamma inflows. J Hydrol 1993;146:383–9.
- [34] Prabhu NU. Some exact results for the finite dam. Ann Math Statist 1958;29:1234–43.
- [35] Pretto RM, Chiew FHS, McMahon TA, Vogel RM, Stedinger JR. The (mis)behavior of behavior analysis storage estimates. Water Resour Res 1997;33(4):703–9.
- [36] Rippl W. Capacity of storage reservoirs for water supply. Minutes of Proc, the Inst Civil Eng 1883;71:270–8.
- [37] Simonovic SP. Sustainability criteria for possible use in reservoir analysis. In: Takeuchi K, Hamlin M, Kundzewicz ZW, Rosbjerg D,

- Simonovic I, editors. Sustainable reservoir development and management. International Association of Hydrological Sciences Publishers. p. 251.
- [38] Sudler CE. Storage required for the regulation of stream flow. Trans Am Soc Civil Eng 1927;61:622–60.
- [39] Thomas HA, Burden RP. Operations research in water quality management. Division of Engineering and Applied Physics, Harvard University; 1963.
- [40] Troutman BM. Limiting distributions in storage theory, PhD thesis, Colorado State University, Fort Collins, Colorado; 1976.
- [41] Vogel RM, Bolognese RA. Storage-reliability-resilience-yield relations for overyear water supply systems. Water Resour Res 1995;31(3):645–54.

- [42] Vogel RM, Fennessey NM. L-moment diagrams should replace product moment diagrams. Water Resour Res 1993;29(6):1745–52.
- [43] Vogel RM, McMahon TA. Approximate reliability and resilience indices of over-year reservoirs fed by AR(1) Gamma and normal flows. Hydrol Sci J 1996;41(1):75–96.
- [44] Vogel RM, Stedinger JR. Generalized storage-reliability-yield relationships. J Hydrol 1987;89:303–27.
- [45] Vogel RM, Wilson I. The probability distribution of annual maximum, minimum and average streamflow in the United States. J Hydrol Eng ASCE 1996;1(2):69–76.
- [46] Vogel RM, Lane M, Ravindiran RS, Kirshen P. Storage reservoir behavior in the United States. J Water Resour Planning Manage 1999;125(5):245-54.