

One possible solution

1. The geoid is an equipotential surface of the Earth's gravity field.
 - (a) Which is the natural surface that is approximated by the geoid?
The mean sea level (known as MSL).
 - (b) What is the “geoid undulation”?
If H the distance of a location to the geoid (MSL elevation) and h is the distance to the Earth ellipsoid (geodetic elevation), the geoid undulation is $N=H-h$.
 - (c) What is, approximately, the largest absolute value of the geoid undulation for the WGS84 datum?
100m approximately.
 - (d) How do local GPS elevation measurements depend on the geoid undulation?
GPS elevation readings can be with respect to the ellipsoid (geodetic) or with respect to the geoid (MSL). The difference between them is precisely the geoid undulation.
2. The MODIS' Climate Modeling Grid (CMG) datasets provide global coverage in a geographic lat/long coordinates at a resolution of 0.05° .
 - (a) Explain why the surface on the ground is not the same for every grid cell.
Since the distance on the ground for 1° of longitude depends on the latitude and since the cells have all the same size in degrees, then the surface for each cell varies with the latitude.
 - (b) Show that a complete coverage of the Earth's surface requires 25920000 grid cells.
Each $1^\circ \times 1^\circ$ tile has $20 \times 20 = 400$ CMG grid cells. For the whole surface of the Earth there are $360 \times 180 \times 400 = 25920000$ grid cells.
 - (c) What is approximate size (in meters) of a cell grid on the surface along a meridian (Consider the spherical model for the Earth with radius 6371km)?
Since $0.05^\circ = \pi \cdot 0.05 / 180 = 0.00087266462 \text{ rad}$, then the size is $0.00087266462 \times 6371000 = 5559.746 \text{ m}$.
3. Consider the following coordinate reference system (CRS) EPSG:3021 described by PROJ.4 parameters:

```
+proj=tmerc +lat_0=0 +lon_0=15.80827777777778 +k=1  
+x_0=1500000 +y_0=0 +ellps=bessel  
+towgs84=414.1,41.3,603.1,0.855,-2.141,7.023,0  
+units=m +no_defs
```

 - (a) What is the meaning of the `+proj=tmerc` parameter?
It indicates that the map projection is Transverse Mercator.
 - (b) Where does this projection lead to lower distortions? In other words, in which part of the world is this CRS likely to be used?
The parameters `+lat_0=0, +lon_0=15.80827777777778` indicate the origin of the projection, where the distortions vanish. For the Transverse Mercator projection, the errors vanish along the 15.80827777777778° meridian (Obs: which crosses Europe and Africa, from Sweden to Angola).
 - (c) What is the meaning of the `+x_0=1500000` parameter?
It is the false easting of the projected coordinates.
 - (d) A user wants to transform coordinates EPSG:3021 into ETRS89–LAEA coordinates. Explain why this transformation of coordinates involves a datum transformation.
Because the datums are distinct. This is clear since the description of EPSG:3021 contains the parameter `+towgs84` which determines the datum transformation to WGS84, which is considered identical to the datum ETRS89.
 - (e) The map projection in the CRS EPSG:3021 is “conformal”. If you apply it to the lat/long coordinates that define the boundaries of a given town, which characteristic is going to be preserved in the resulting planar map: shape, total area, or distance of the boundary to the center of the town?
The shape.

4. Suppose you want to georeference the following aerial photograph into the WGS84 CRS.



- (a) Which technique would you use? What information do you need to apply it?

The technique to use is georegistration with control points. To apply it one needs, for each control point, its coordinates on the image (row, column of the image) and also its WGS84 coordinates (lat/long).

- (b) Which kind of coordinate transformation could you fit given the features available in the photograph?

There are many features well distributed over the image that could possibly be used as control points (road intersections and corner of agriculture fields in particular). Therefore, one can possibly define a dozen control points, which would suffice for a 1st or 2nd degree transformation of coordinates.

- (c) What is “nearest neighbor resampling” in this context? Explain using a formal notation.

The georegistration technique would create an empty grid in WGS84 coordinates. Consider an arbitrary grid cell with center $(\bar{x}_{\text{ref}}, \bar{y}_{\text{ref}})$. This center is mapped by the estimated transformation function into a location $(\bar{x}_{\text{transf}}, \bar{y}_{\text{transf}})$ that lies somewhere over the aerial photograph. The nearest neighbor criteria associates to the grid cell with center $(\bar{x}_{\text{ref}}, \bar{y}_{\text{ref}})$ the value of the aerial photograph pixel closest to $(\bar{x}_{\text{transf}}, \bar{y}_{\text{transf}})$. Applying this procedure to all grid cells completes the georegistration process.