

One possible solution

1. Consider the following digital elevation model (DEM) data set with a 10m resolution and the derived slope raster data set.

DEM (resolution=10m)										Slope (rounded to 2 decimals)									
15	11	8	8	9	13	18	21	23	24										
19	15	11	9	10	13	17	21	22	24		0.51	0.43	0.21	0.17	0.36	0.41	0.26	0.17	
21	19	15	12	10	12	16	20	21	23		0.42	0.48	0.38	0.16	0.25	0.38	0.3	0.2	
23	21	19	16	13	13	14	18	20	22		0.32	0.39	0.43	0.32	0.15	0.25	0.29	0.25	
25	23	21	19	17	14	15	16	18	20		0.28	0.3	0.34	0.36	0.22	0.11	0.19	0.23	
27	25	23	21	20	17	15	16	17	19		0.27	0.28	0.27	0.32	0.32	0.16	0.08	0.16	
28	27	25	23	22	20	18	16	17	18		0.25	0.28	0.25	0.27	0.34	0.31	0.16	0.06	
30	29	27	25	24	23	21	19	17	17		0.25	0.28	0.27	0.28	0.31	0.34	0.31	0.15	
32	31	29	28	27	25	24	22	19	18		0.25	0.27	0.26	0.28	0.3	0.33	0.36	0.31	
34	33	31	30	29	28	27	24	23	20										

- (a) Write an expression using arithmetical and logical operations to reclassify the DEM raster data set according to the following table:

current value	new value
elevation less than 21m	0
elevation equal to 21 m	1
elevation higher than 21m and less than 25.5m	2
elevation equal or higher than 25.5m	3

One solution:

$0 * (\text{DEM} < 21) +$
 $1 * (\text{DEM} = 21) +$
 $2 * (\text{DEM} > 21 \text{ AND } \text{DEM} < 25.5) +$
 $3 * (\text{DEM} \geq 25.5)$

- (b) Estimate the flow direction at the gray pixel;

One solution: The flow direction is given by the vector $(-S_x, -S_y)$ where S_x is the estimate of the slope along the x direction and S_y is the estimate of the slope along the y direction. The usual estimates of S_x and S_y are:

$$S_x = \frac{(21-13)+2 \times (20-12)+(18-13)}{8 \times 10} = 0.3625$$

$$S_y = \frac{(13-13)+2 \times (17-14)+(21-18)}{8 \times 10} = 0.1125$$

Therefore, the estimated flow direction is given by the vector $(-0.3625, -0.1125)$ that points towards the third quadrant (since both coordinates are negative).

- (c) Estimate the aspect of the surface at that pixel (if you haven't answered the previous question, consider that the estimates of the slope along the X direction and the y direction are, respectively, $S_x = 0.35$ and $S_y = 0.1$);

One solution: The aspect is the flow direction given by its azimuth (from North, clockwise). Since the flow direction points towards the 3rd quadrant, then the azimuth has to be between 180° and 270° . More precisely it is $\arctan(S_x/S_y) + 180^\circ = \arctan(0.3625/0.1125) + 180^\circ = 72.758^\circ + 180^\circ = 252.758^\circ$. Its direction is W.

- (d) Apply a uniform moving window with the following kernel

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

to the DEM at the location in gray. In general, what is the effect of applying a uniform moving window to a raster data set?

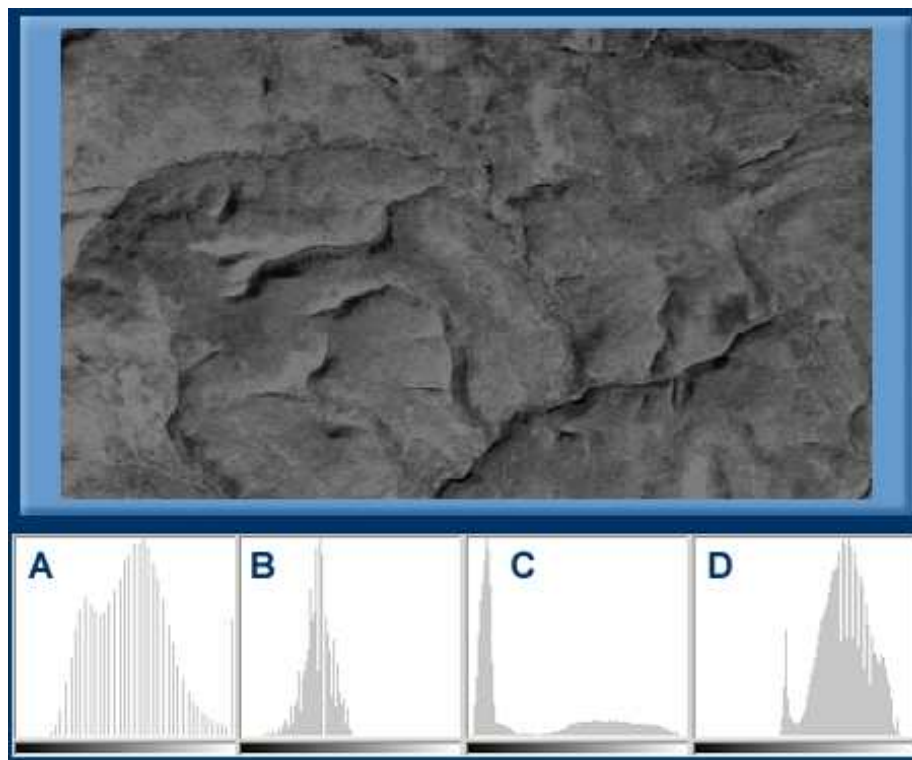
One solution: The result of the application of the moving window to the pixel in gray returns

$$\frac{1}{9} \times 13 + \frac{1}{9} \times 17 + \frac{1}{9} \times 21 + \frac{1}{9} \times 12 + \frac{1}{9} \times 16 + \frac{1}{9} \times 20 + \frac{1}{9} \times 13 + \frac{1}{9} \times 14 + \frac{1}{9} \times 18 = 16.$$

Since the window is uniform (i.e. all the weights of the kernel are equal) and the weights are $\frac{1}{9}$ for a kernel with 9 cells, then this is just the average of the window values. In general, the application of a moving window like this one is going to *smooth* the input image.

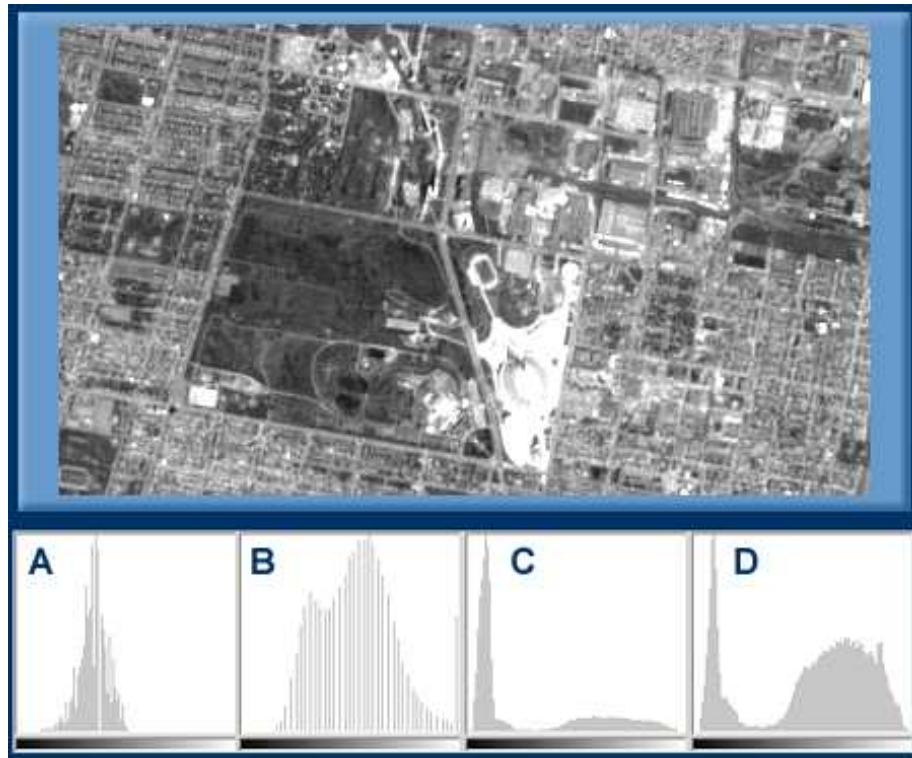
2. Consider the following raster data sets. Indicate the correct histogram for each one and explain your choice. Note that the x axis of the histogram indicates *increasing* cell values (from left to right) and the y axis indicates frequencies (number of pixels) .

- (a) Raster 1:



One solution: The answer is B, since it is a mostly dark gray image, with low contrast. A would correspond to an raster with the whole range of digital numbers, for 0 to the maximum, with a saturation for bright pixels (see following problem). C corresponds to an image with mostly very dark pixels (very low digital numbers) and some bright pixels, up to to the maximum digital number. D would correspond to a bright image.

(b) Raster 2:



One solution: The answer is B since it is the histogram of a raster with the whole range of digital numbers, for 0 to the maximum, with a saturation for bright pixels. A is the histogram of the previous problem. C corresponds to an image with mostly very dark pixels (very low digital numbers) and some bright pixels, up to to the maximum digital number. D would correspond to an image with approximately 1/4 of very dark pixels, and 3/4 of bright and very bright pixels, with a gap in between.

- (c) Which one of the above images is likely to benefit from a linear contrast enhancement? (if you have answered to the previous questions, use your answers explain your choice; otherwise explain how a linear contrast enhancement would modify the intensity of the images, and choose the image that would benefit the most from that technique by inspecting visually each of them).

One solution: The image that would benefit the most from linear enhancement would be the image from problem (a). If we call $z_{\min}$ the value associated to the left tail of the histogram, and $z_{\max}$ the value associated to the right tail of the histogram, the contrast of the image could be enhanced by given the lowest intensity to $z_{\min}$ and the maximum intensity to $z_{\max}$. Applying the same procedure to the image of problem (b), would not change significantly the intensity of its pixels (and therefore its contrast).

3. Whenever a GPS receiver is turned on, a synchronization process starts. The user must wait until the synchronization process is over before gathering data. Explain why prior synchronization is necessary to obtain accurate positional data.

One solution: On GPS satellites, timing is almost perfect since satellites use precise atomic clocks. Atomic clocks are too expensive to be used by GPS receivers. Therefore, GPS receivers do a fourth measurement (in addition to the three measurements that would be sufficient to determine the position if the clocks were perfectly synchronized) to determine a correction factor that can be applied to its own (imprecise) timing measurements. This is called the “the synchronization process”.